Linear Stability Analysis of the Reacting Shear Flow

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The linear instability of reacting shear flow is analyzed with special emphasis on the effects of the heat release and variable transport properties. Both analytic profiles and laminar solutions of the boundary-layer equations are used as base flows. The growth rates of the instabilities are sensitive to the laminar profiles, differing by more than a factor of 2 according to which profile is used. Thus, it is important to base the analysis on accurate laminar profiles. Accounting for variable transport properties also changes the mean profiles considerably, and so including them in the computation of the laminar profiles is equally important. At larger heat release, two modes that are stronger in the outer part of the shear layer have the highest growth rates ; they also have shorter wavelengths than the center mode.

Key Words : Instability, Reacting Shear Flow

1. Introduction

Prediction of the characteristics of chemically reacting mixing layers is very important in a number of technologies. For increased mixing, we prefer to have turbulent flow, which occurs only if the laminar flow is unstable. Therefore, it is important to analyze the stability of reacting shear layers ; linear stability analysis is the most convenient tool for this purpose.

The stability of nonreacting mixing layers has been extensively investigated. In most of this work, analytic mean velocity profiles (usually hyperbolic tangent or error functions) were used. The validity of doing so needs investigation.

For incompressible parallel inviscid flow, Rayleigh (1880) showed that, if the velocity profile has an inflection point, the flow is unstable. Lin (1955) suggested that the inviscid mechanism dominates at large Reynolds numbers with viscosity producing only slight damping. Michalke (1964) numerically integrated the Rayleigh stability equation with the hyperbolic-tangent velocity profile for temporally as well as spatially growing disturbances to incompressible flow ; the spatial case results agreed well with experiments.

The effects of the mean velocity profile were studied by Monkewitz and Huerre (1982), who found that the amplification rate found with the Blasius mixing layer velocity profile agreed well with experimental results. Morkovin (1988) suggested that only stability analysis based on mean profiles derived from the boundary-layer equations should be compared with experimental re-

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sults ; this is consistent with Michalke's proposition.

Trouve and Candel (1988) did linear stability analysis of the inlet jet in a ramjet dump combustor using hyperbolic-tangent velocity and temperature profiles. They found that the density variation has a significant effect on the instability. Recently, Mahalingam et al. (1989) studied the effects of heat release on the stability of co-flowing, chemically reacting jets. They suggested that the heat release due to chemical reaction stabilized the flow.

For compressible flow, Sandham and Reynolds (1989) solved the linearized inviscid compressible stability equation and found maximum amplification at the frequency at which vortices are found in the laboratory. They also found that three-dimensional effects are important at high Mach number.

In this paper, we consider a low-speed, plane shear layer in which fuel and oxidizer are initially unmixed. Figure 1 shows the schematic diagram of spatially developing shear layer. Laminar profiles obtained by solving the boundary-layer equations are used as input to linear stability analysis. The effects of heat release in both the laminar flow and the instability are studied as are the effects of variable transport properties. Both temporally and spatially developing layers are considered ; the former are easier to understand, and the latter are used for comparison with experimental results.

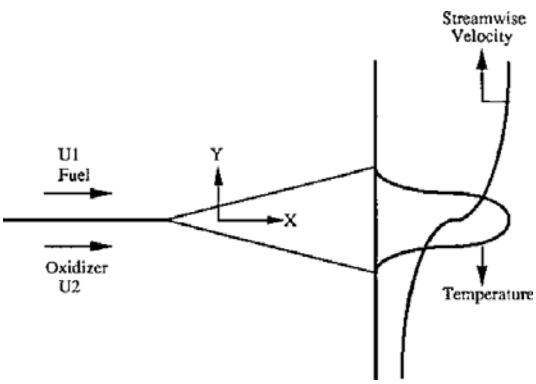


Fig. 1 Schematic diagram of spatially developing shear layer

2. Laminar Flow Profiles

To generate mean profiles, we solved the (parabolic) two-dimensional boundary-layer equations in the low Mach number approximation. The effects of density variation, which can be quite large if the heating caused by chemical reaction is large, must be retained. All dependent variables are nondimensionalized using the values on the high-speed side. Uniform pressure through the shear layer and unity Lewis and Prandtl numbers are assumed for simplicity. We found that a Prandtl number of 0.7 does not produce large quantitative differences. Both freestreams are at the same temperature. The inlet profiles are taken from self-similar solutions of the equations for the time developing flow at low heat release but the temperature is arbitrarily increased so that reaction will proceed. Chemistry is represented by a single step irreversible scheme, involving fuel F and oxidizer O, reacting to yield a product P :

$$v_F F + v_O O \rightarrow v_P P \tag{1}$$

where v_F, v_O, and v_P are the stoichiometric coefficients for fuel, oxidizer, and product, respectively. Both constant and variable property cases are simulated. For the variable properties, power laws in temperature and pressure are assumed :

$$\mu \propto P^0 T^{0.7},\ \kappa \propto P^0 T^{0.7},\ D \propto P^1 P^{1.7} \tag{2}$$

where μ is the viscosity, κ the thermal diffusivity, and D the mass diffusivity. The non-dimensional adiabatic flame temperature, T_{ad}, is related to the heat of reaction Q as follows :

$$T_{ad} = Q + 1 \tag{3}$$

To satisfy the boundary-layer approximation, we used an initial Reynolds number of 1×10^3 based on the vorticity thickness and the cold viscosity. The vorticity thickness of the initial velocity profile, δ_ω, is used as the reference length scale :

$$\delta_\omega = \frac{U_1 - U_2}{|du/dy|_{\max}} \tag{4}$$

To include coupling between chemical reaction, heat release, and the velocity field, the continuity, momentum, energy, and species equations are solved simultaneously. As the boundary-layer equations are parabolic, an implicit method (Crank-Nicolson) is used. In the spatial layer, the correct boundary condition for the normal velocity V as $y \to \infty$ is difficult to determine. We used the integral relation derived from the y-momentum equation :

$$\frac{d}{dx}\int_{-\infty}^{\infty}\rho U V dy + \rho_{\infty} V_{\infty}^2 - \rho_{-\infty} V_{-\infty}^2 = 0 \qquad (5)$$

where ρ is the density and U the streamwise velocity.

The magnitude of the normal velocity is $<1\%$ of the streamwise velocity in all computed profiles ; this validates the use of the boundary-layer approximation.

Figures 2 compare the streamwise velocity and temperature profiles of the spatial layer for the same inlet profile and show the effects of heat

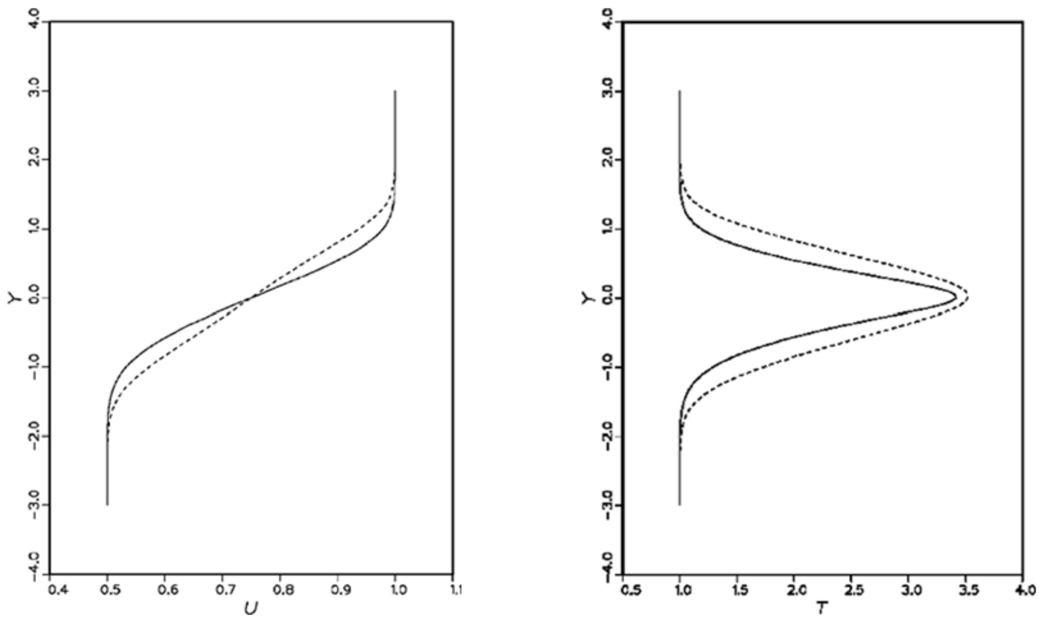


Fig. 2 Effect of variable properties of the spatially developing layer for : (a) velocity (b) temperature. ———, constant property ; ---------, variable property

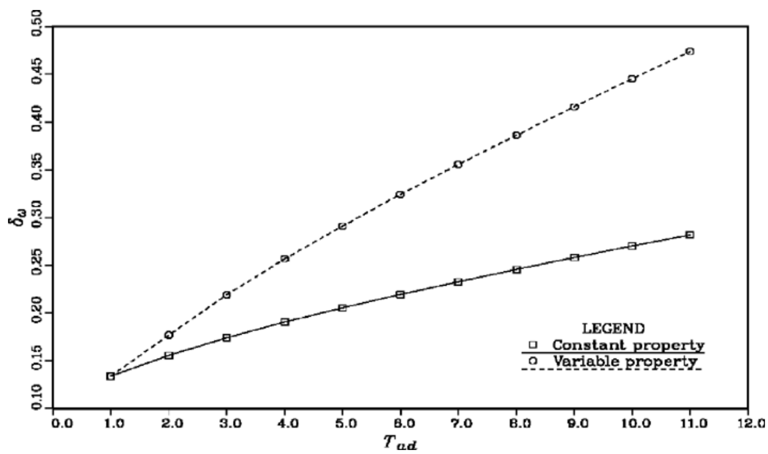


Fig. 3 Effect of heat release to the growth of the velocity thickness of spatially developing layer

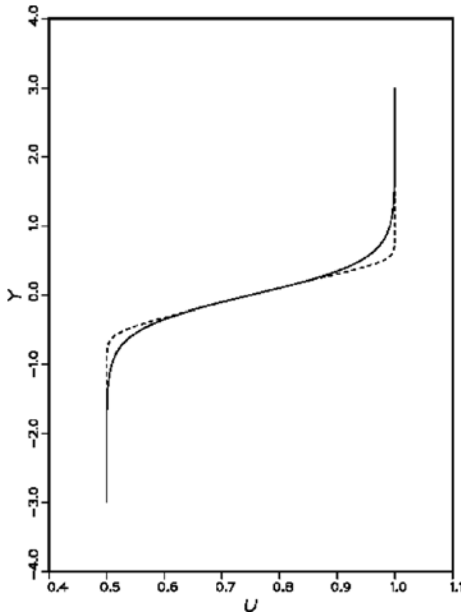


Fig. 4 Comparison of the hyperbolic-tangent and laminar solution velocity profiles for the spatially developing layer at $T_{ad}=8$. ———, hyperbolic-tangent function ; --------, laminar solution

release and variable properties on the laminar flow. These profiles are compared at the same nondimensional streamwise distance x for downstream from the ignition point. The nondimensional adiabatic flame temperature T_{ad} is 8 for the reacting cases. Figure 3 shows that the vorticity thickness δ_ω with variable properties grows more rapidly as the adiabatic flame temperature increases. The significant difference between the constant and variable property solutions indicates that property variations need to be included whenever there are large temperature variations.

Figure 4 compares the laminar solution with a hyperbolic-tangent profile of the same vorticity thickness. The profile obtained from the simulation is fuller than the hyperbolic-tangent profile and, as we shall see, has very different stability properties.

3. Linear Stability Analysis

In linear inviscid stability analysis, all flow variables are assumed to be the sum of the mean and small wave-like fluctuations. The parallel flow assumption is made for the mean flow, which means that the predominant variations of the mean flow properties vary in the direction normal to the flow. All flow variables can be represented in the form :

$$f(x, y, z, t)=\bar{f}(y)+f'(x, y, z, t) \qquad (6)$$

The disturbances represented by the primed variables are assumed to have the form of traveling waves :

$$f'(x, y, z, t)=\hat{f}(y)\exp[i(\alpha x+\beta z-\omega t)] \qquad (7)$$

where $\hat{f}(y)$ is an eigenfunction assumed to be a function of y only, α and β are wave numbers in the streamwise and spanwise directions, respectively, and ω is the frequency. The relation between the wave numbers and the angle of disturbance is

$$\tan\theta=\beta/\alpha \qquad (8)$$

For the temporal stability analysis, α is real and ω is complex, whereas for the spatial analysis, ω is real and α is complex. The amplification rates for the two cases are ω_i and $-\alpha_i$, respectively.

The perturbation equations are derived by linearizing the low Mach number equations without using the boundary-layer approximations. Substituting Eq. (6) into these equations and neglecting the products of disturbances yields the equations for the perturbations. From the continuity and momentum equations, a disturbance equation for the pressure is obtained :

$$\hat{p}''-\frac{2\alpha U'}{\alpha U-\omega}\hat{p}'+\hat{\rho}(\alpha U-\omega)^2-(\alpha^2+\beta^2)\hat{p}=0 \qquad (9)$$

where U is the mean velocity and a prime denotes differentiation with respect to y. This is the three-dimensional equation ; for $\beta=0$, it reduces the two-dimensional one. Equation (9) reduces to the incompressible Rayleigh equation (Drazin, 1982) if density is constant, i.e., $\hat{\rho}=0$. Finally, $\hat{\rho}$ can be eliminated in favor of the mean pressure $\hat{p}$ using the state, energy, and species equations. Equation (9) then becomes

$$\hat{p}''-\left\{\frac{2\alpha U'}{\alpha U-\omega}+\frac{R}{T}(\alpha U-\omega)^2[RXN]\right\}\hat{p}' -(\alpha^2+\beta^2)\hat{p} \qquad (10)$$

where R and T are the mean density and temperature, respectively. $[RXN]$ is a term that represents the effect of density variation due to chemical reaction and associated heat release. The boundary conditions are obtained by considering the asymptotic form of the solutions of Eq. (10). As $y \to \pm\infty$, U' and $[RXN]$ become negligible and the bounded solutions behave like

$$\hat{p}(y \to -\infty) = C_1 \exp(\sqrt{\alpha^2+\beta^2}\,y) \qquad (11a)$$

$$\hat{p}(y \to \infty) = C_2 \exp(\sqrt{\alpha^2+\beta^2}\,y) \qquad (11b)$$

where C_1 and C_2 are arbitrary constants. A combination of the shooting and Newton-Raphson methods are used to solve Eq. (10). This method is applicable to both the temporal and spatial problems. Any velocity and temperature profiles can be specified as input to the stability calculation ; in particular, either analytic functions or the computed laminar profiles can be used.

According to Rayleigh's inflection point theorem (Drazin, 1982), a necessary condition for instability is that the laminar velocity profile has an inflection point. For incompressible flow, this condition requires U'' to change sign at least once in the flow domain. A stronger form of this condition was obtained by Fjørtoft (Drazin, 1982), who proved that a necessary condition for instability is that $U''(U-U_s)<0$ somewhere in the field, where y_s is a point at which $U''=0$ and $U_s=U(y_s)$. Here, we generalize these theorems to flows that have density variation.

If we write the disturbance equation in terms of the normal fluctuating velocity $\hat{v}$, then, in temporal flow, the disturbance equation becomes

$$\left(\frac{\hat{v}'}{R}\right)' - \left\{\frac{(RU')'}{R^2(U-C)} + \frac{\alpha^2}{R}\right\}\hat{v} = 0 \qquad (12)$$

where C is the complex wave velocity. Multiplying this equation by $\hat{v}^*$, integrating from $-\infty$ to ∞, and integrating the first term by parts, we obtain

$$\int_{-\infty}^{\infty}\left\{\frac{|D\hat{v}|^2+\alpha^2|\hat{v}|^2}{R} + \frac{(U-C_r)(RU')|\hat{v}|^2}{R^2|U-C|^2}\right\}dy + ic_i\int_{-\infty}^{\infty}\frac{(RU')'|\hat{v}|^2}{R^2|U-C|^2}dy = 0 \qquad (13)$$

where D is the differential operator d/dy, and C_r and C_i are the real and imaginary parts of the wave velocity, respectively. The imaginary part of Eq. (13) is

$$C_i\int_{-\infty}^{\infty}\frac{(RU')'|\hat{v}|^2}{R^2|U-C|^2}dy = 0 \qquad (14)$$

which can be satisfied only if $(RU')'$ changes sign at least once in the open domain $(-\infty, \infty)$; this is a necessary condition for instability. A stronger form of this condition can be obtained by considering the real part of Eq. (13):

$$\int_{-\infty}^{\infty}\frac{(U-C_r)(RU')|\hat{v}|^2}{R^2|U-C|^2}dy = -\int_{-\infty}^{\infty}\frac{|D\hat{v}|^2+\alpha^2|\hat{v}|^2}{R}dy \qquad (15)$$

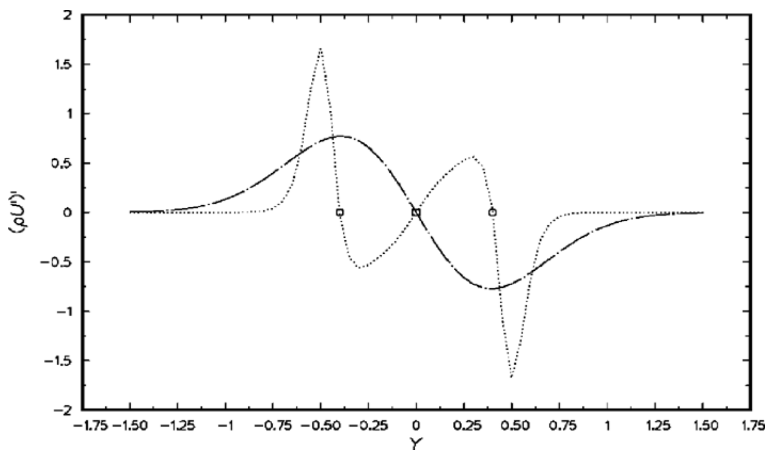


Fig. 5 The function appearing in the necessary condition for temporal instability. $-\cdot-\cdot-$, $T_{ad}=1$; ··········, $T_{ad}=8$

Supposing that $C_i>0$, multiplying Eq. (14) by $(C_r-U_s)/C_i$ and adding it to Eq. (15), we have

$$\int_{-\infty}^{\infty}\frac{(U-U_s)(RU')|\hat{v}|^2}{R^2|U-C|^2}dy = -\int_{-\infty}^{\infty}\frac{|D\hat{v}|^2+\alpha^2|\hat{v}|^2}{R}dy \tag{16}$$

Thus, a necessary condition for instability is $(RU')'(U-U_s)<0$ somewhere on the domain $-\infty<y<\infty$. The mean profiles were searched for points that satisfy this condition. Figure 5 shows that the cold flow has just one inflection point, whereas the reacting flow has three. Figure 6 shows a test of the strong necessary condition for instability. All three inflection points satisfy the necessary condition. The reacting shear layer should, therefore, be unstable to three distinct modes.

We showed that the mean profiles for reacting flow have three inflection points and should have three independent modes of instability. In the temporal stability case, symmetry dictates that two of these be reflections of each other. Figure 7 show the amplification rate and phase velocity as a function of the wave number for a variable property flow with $T_{ad}=8$. For the amplification rate, only two modes are shown. The first is the center mode that arises from the central inflection point; its phase velocity is the mean velocity at the central inflection point. The second represents both of the modes due to the outer inflection points. The phase velocities of the two outer modes are different, as shown in Fig. 7(b), and they approach the mean velocities at the outer inflection points as the wave number increases.

The center mode travels at the same phase velocity as in the cold flow, the average of the two freestream velocities. One of the outer modes travels at lower speed than the center mode, whereas the outer travels at higher speed. Similar results were found in compressible flow by Sandham (1989).

We shall show that, for large heat release, the outer modes are more amplified and should dominate. These modes are also very sensitive to the variation of the properties, and so the latter is very important in this flow. Figure 8 shows the effect of heat release on the maximum growth rate for the spatially developing layer; computed laminar profiles were used in these calculations. As the heat release increases, the maximum amplification rate of the center mode decreases. The maximum amplification rate in the cold flow is 0.128, but for the variable property case with T_{ad}, it is 0.01, or only 8% that of the cold flow. On the other hand, the amplification rate of the outer mode changes very little as the heat release increases. Consequently, at high heat release, the other mode is more amplified than the center mode. When T_{ad} is 10, the outer mode has almost

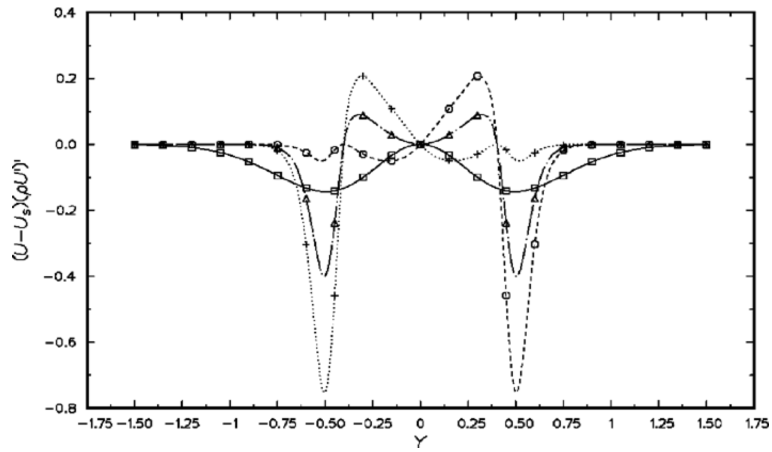
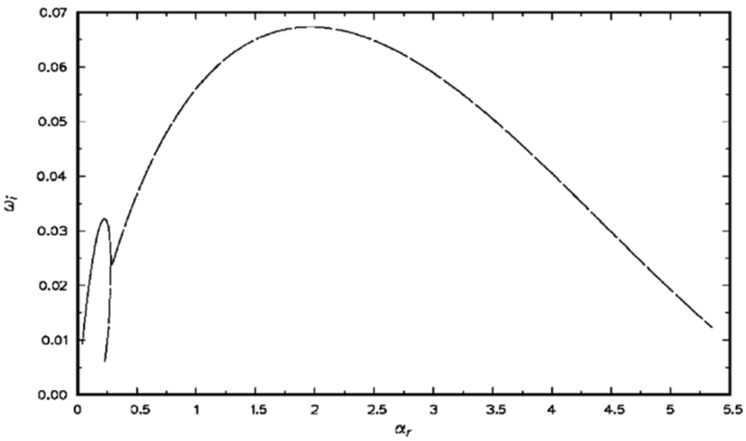
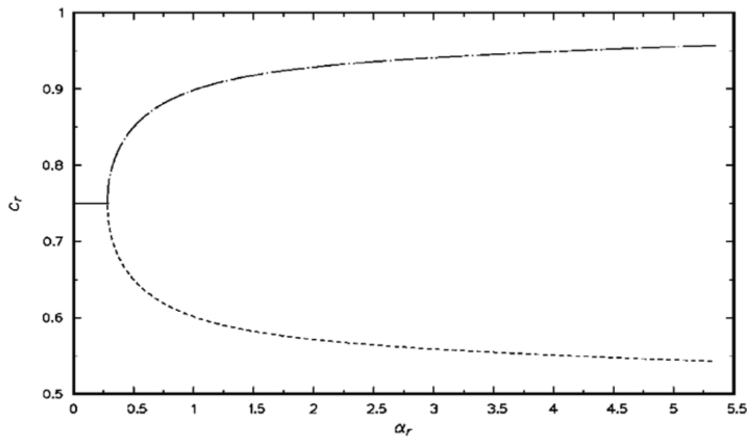


Fig. 6 The function appearing in the strong necessary condition for temporal instability. □, $T_{ad}=1$, $\bar{u}_s=0.75$; ○, $T_{ad}=8$, $\bar{u}_s=0.64$; △, $T_{ad}=8$, $\bar{u}_s=0.75$; +, $T_{ad}=8$, $\bar{u}_s=0.96$



(a)



(b)

Fig. 7 The multiple instability modes in the temporal flow. (a) growth rate (b) phase velocity. ———, center mode ; ---------, slow mode ; – · – · –, fast mode. Note that the two outer modes have identical growth rates

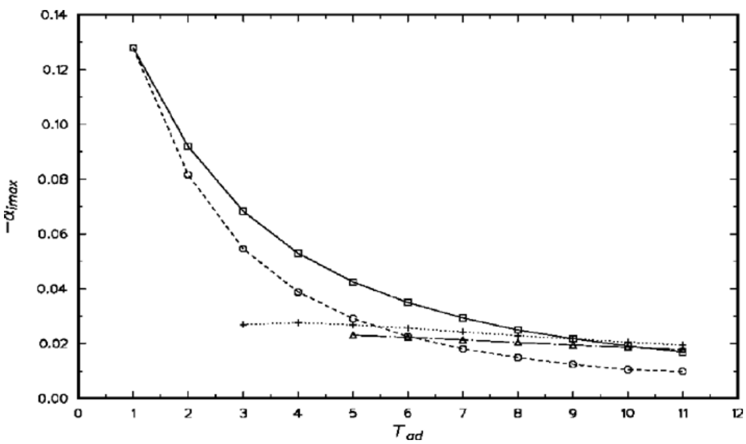


Fig. 8 Effect of heat release on the amplification rate (spatial instability). □, center mode, constant property ; ○, center mode, variable property ; △, slow mode, constant property ; +, slow mode, variable property

three times the amplification rate of the center mode. Thus, reacting flows with high heat release should be unstable to the short wavelength outer modes.

The absolute and convective instability of the spatially developing layer are distinguished by the temporal growth rate, which is positive in the absolute and negative in the convective case, of the mode that dominates the response at the source location. In physical terms, in absolute instability, a locally generated small amplitude transient grows exponentially in time, whereas in convective instability, transient is convected away and leaves the mean flow ultimately undisturbed. Huerre and Monkewitz (1985) showed that a flow is convectively unstable if the modes that have zero group velocity are temporally damped, i.e., the imaginary parts of the complex frequencies are negative and absolutely unstable if all are positive. We used this criterion to determine the nature of the instability of the reacting mixing layer. First, we found the complex frequency ω_o, which makes the group velocity $d\omega/d\alpha$ zero. The imaginary part of this frequency ω_o is the absolute growth rate that determines the nature of the instability. Figure 9 shows the imaginary part of ω_o as a function of the adiabatic flame temperature T_{ad}. All ω_{oi} are negative, and, therefore, the reacting mixing layers considered here are convectively unstable.

We showed that the variation of the properties through the reacting shear layer influences the mean flow profiles significantly. Figure 10 shows that that effect results in the difference in stability characteristics when $T_{ad}=8$. The constant property case has twice the growth rate of the center mode of the variable property case ; however, the latter has the growth rates of the outer modes, which are almost 25% higher. The constant property profile has the center and outer modes with comparable amplification rates, but for the variable property profile, the outer modes are dominant over the center mode. Again, the importance of the variable properties is emphasized.

Squire's theorem (Drazin, 1982) states that the lowest Reynolds number for transition occurs when the disturbances are two-dimensional, so two-dimensional modes dominate the viscous instability of incompressible flows. The Reduction of a three-dimensional problem to a two-dimensional one lowers of the order of the system of equation, reduces the number of integrations to find the eigenvalue and reduces this size of the parameter space that must be investigated. Once the eigenvalue is known, calculation of the eigenfunction requires as many integrations as these are components of the velocity. In inviscid problems (infinite Reynolds number), the transformation is of some help, but the advantages of Squire's theorem may not be obtained.

In compressible flows, the most amplified modes are three-dimensional (Sandham and Reynolds,

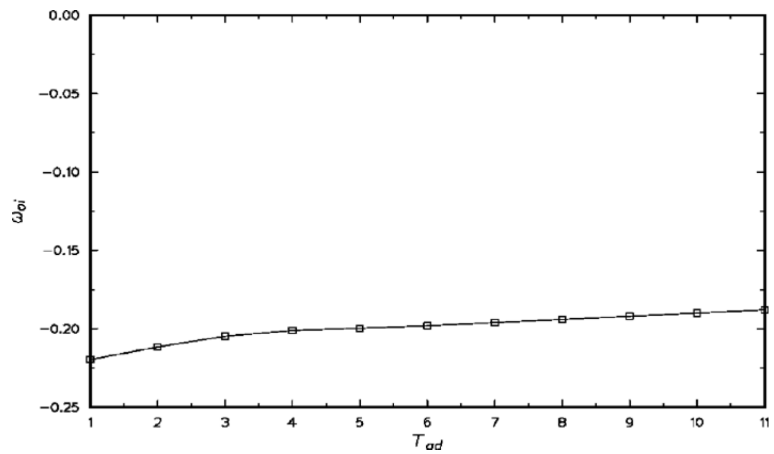


Fig. 9 Variation of ω_{oi} with adiabatic flame temperature (spatial instability)

1989). We tried to determine whether the most amplified modes are two- or three-dimensional in the incompressible low-speed reacting mixing shear layer. Fig. 11 shows that the disturbances become more stable as the obliquity increases. Therefore, two-dimensional modes are more amplified than three-dimensional ones and three-dimensionality is not important in this case.

Here, the validity of using analytical flow profiles such as hyperbolic-tangent and error functions is investigated. To accomplish this goal, we compare results obtained from these profiles with ones based on the boundary-layer solutions.

The analytical profiles also have three different modes. Figures 12 show the amplification rates vs wave number. In Fig. 12(a), results for temporal layers are given. For the center mode, the differences are small, but for the outer mode, the growth rate obtained from the laminar solution is about twice that obtained with the error function. This is not surprising since the principal differences in the profiles are found in the outer parts of the layers. In Fig. 12(b), the hyperbolic-tangent function results are compared with the laminar solution for the spatial layer. Again, there is little difference for the center mode, but for the outer mode, the hyperbolic-tangent function has a lower growth rate than the laminar

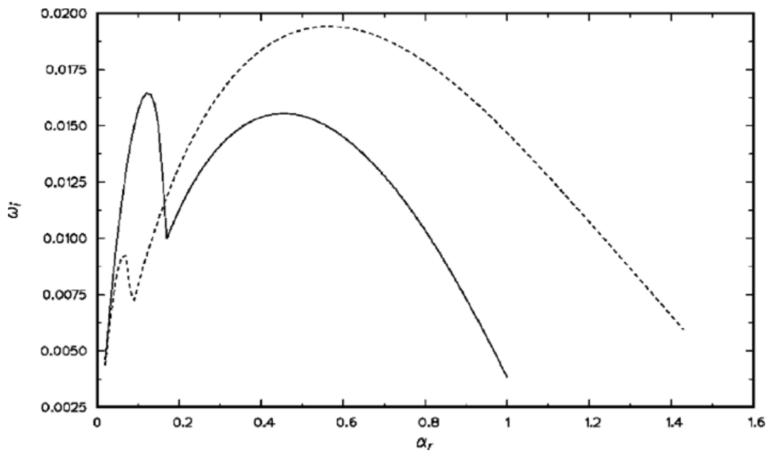


Fig. 10 Effect of variation of properties on the growth rate (temporal instability) $T_{ad}=8$. ———, constant property ; --------, variable property

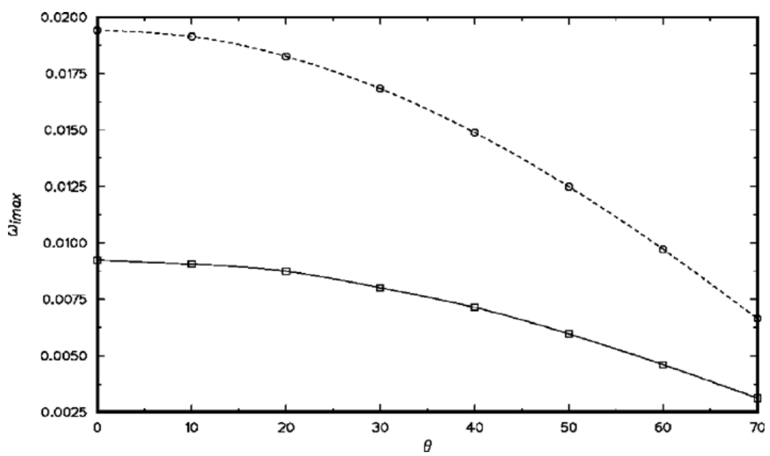


Fig. 11 Growth rates versus obliqueness angle (temporal instability) $T_{ad}=8$. □, center mode ; ○, outer mode

solution. From these results, we find that the use of accurate laminar profiles is essential. This is consistent with the report by Monkewitz and Huerre (1982).

Heat release might be expected to increase the growth rate of the thickness of a mixing layer due to displacement effects caused by dilatation. However, the experiments of Wallace (1981) suggested a slight decrease in the growth rate of reacting mixing layers with increasing heat release. More recently, Hermanson and Dimotakis (1989) conducted with hydrogen and fluorine at $\bar{u}_2=0.4$ and $\bar{T}_2=1$. Fig. 13 compares their experimental data with our computed normalized maximum growth rates. The growth rates are normalized by their values at zero heat release and $\Delta\bar{T}_{max}$ is the maximum temperature rise over the ambient temperature. The maximum amplification rate $|\alpha_i|_{max}$ is linearly related to the growth rate of the shear layer $d\delta/dx$ (Morkovin, 1988; Sandham and Reynolds, 1989). Fig. 13 shows a good qualitative agreement between the linear stability results and the experimental growth rates; the growth rate decrease with increasing heat release. Because of using maximum growth rates instead of the combination of unstable modes, linear stability theory predicts too large a decrease.

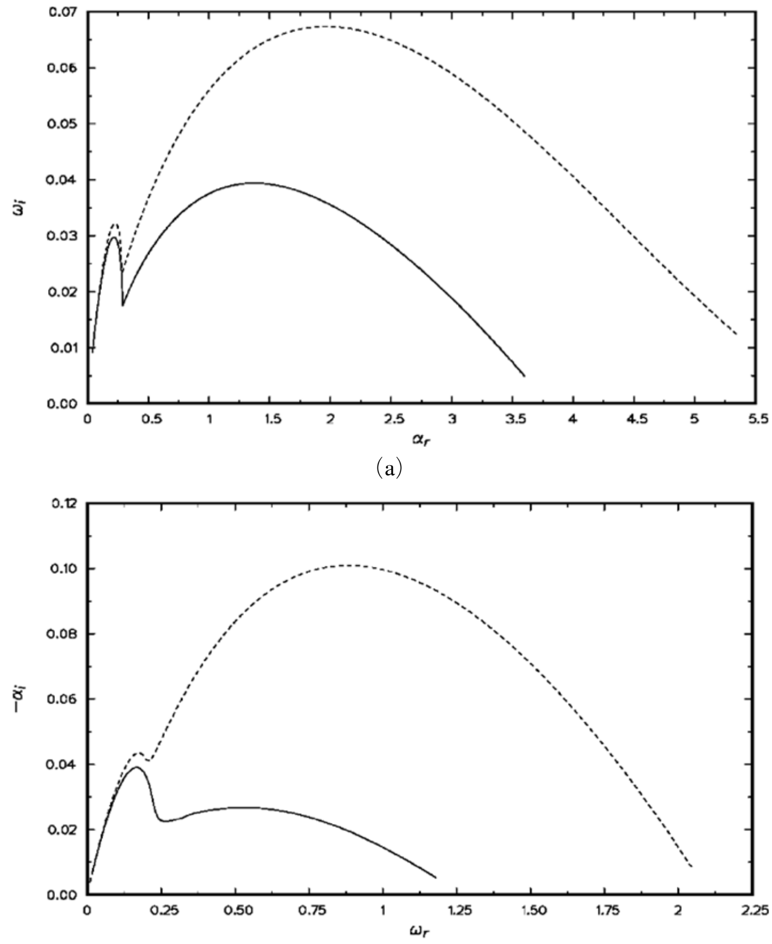


Fig. 12 Effect of velocity profiles on linear growth rates. $T_{ad}=8$. (a) temporal instability. --------, error function; --------, laminar solution. (b) spatial instability. ———, hyperbolic-tangent function; ———, laminar solution

Shin et al. (2005) and Brown and Roshko (1974) identified large spanwise vortices as the principal features of two-dimensional mixing layers in the non-linear region preceding the establishment of fully turbulent flow conditions. The observed initial vortex spacings very nearly correspond to the wavelength of the most amplified mode of instability. We compare the normalized mean vortex spacings measured by Hermanson and Dimotakis (1989) with the wavelengths of the most unstable modes. The wavelengths have been normalized by the wavelengths at zero heat release. To represent the effect of heat release, we used a normalized mean density reduction defined by Hermanson and Dimotakis (1989)

$$\Delta\bar{\rho}/\bar{\rho}_1 = 1 - \bar{\rho}/\bar{\rho}_1 = 1 - \int_{\eta_2}^{\eta_1} \frac{\bar{T}}{\bar{T}_1 + \Delta\bar{T}} d\eta \quad (15)$$

where $\bar{\rho}$ is the integrated mean density in the layer, $\eta_{1,2}$ are the 1% points of the mean temperature profile on the high- and low-speed sides, and $\Delta\bar{T}$ is the temperature rise at each point across the layer. Fig. 14 shows that the wavelengths of the outer modes, which are more unstable than the center modes at high heat release, agree well with the experimental mean vortex spacings. The wavelengths of the center mode increases with increasing heat release.

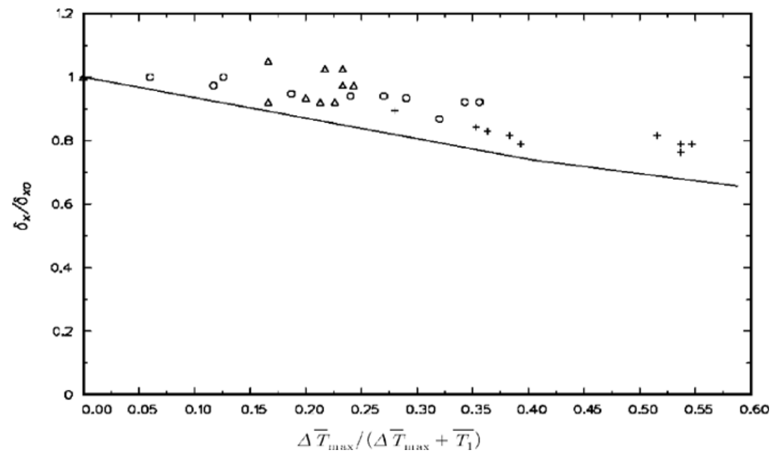


Fig. 13 Normalized growth rate versus heat release. $\bar{T}_2=1$, $\bar{u}_2=0.4$, ——— present ; ○, Wallace (1981) ; △, Mungal (unpublished data); +, Hermanson and Dimotakis (1989)

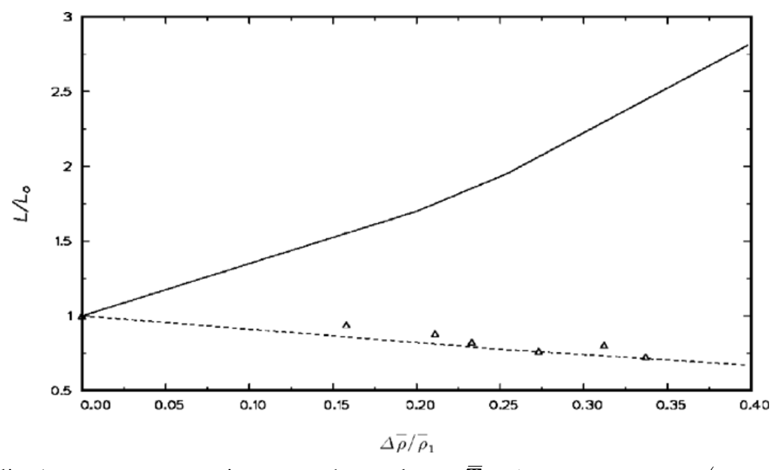


Fig. 14 Normalized mean vortex spacing versus heat release. $\bar{T}_2=1$. ———, present (center mode); ---------, present (outer mode); △, Hermanson and Dimotakis (1989)

5. Summary

We have shown that stability of reacting flows can be studied by linear stability analysis. The growth rates are very sensitive to the mean profiles. Boundary-layer equation solutions obtained with variable transport properties are more realistic representations of an actual flow than analytically prescribed functions and thus provide a better basis for stability analysis. For the reacting plane mixing layer with variable density, a necessary condition for instability has been derived. New inflectional modes of instability are found to exist in the outer part of the mixing layer. Heat release stabilizes the flow and, in particular, greatly reduces the growth rate of the center mode. The growth rates of the outer modes, which do not exist in a cold flow, are relatively insensitive to heat release. For the large heat releases typical of combusting flows, the outer mode is more amplified than the center mode ; its wavelength is shorter than that of the center mode. Even at high heat release, two-dimensional waves are more amplified than three dimensional ones.

Acknowledgments

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